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Print Advising 印刷諮詢
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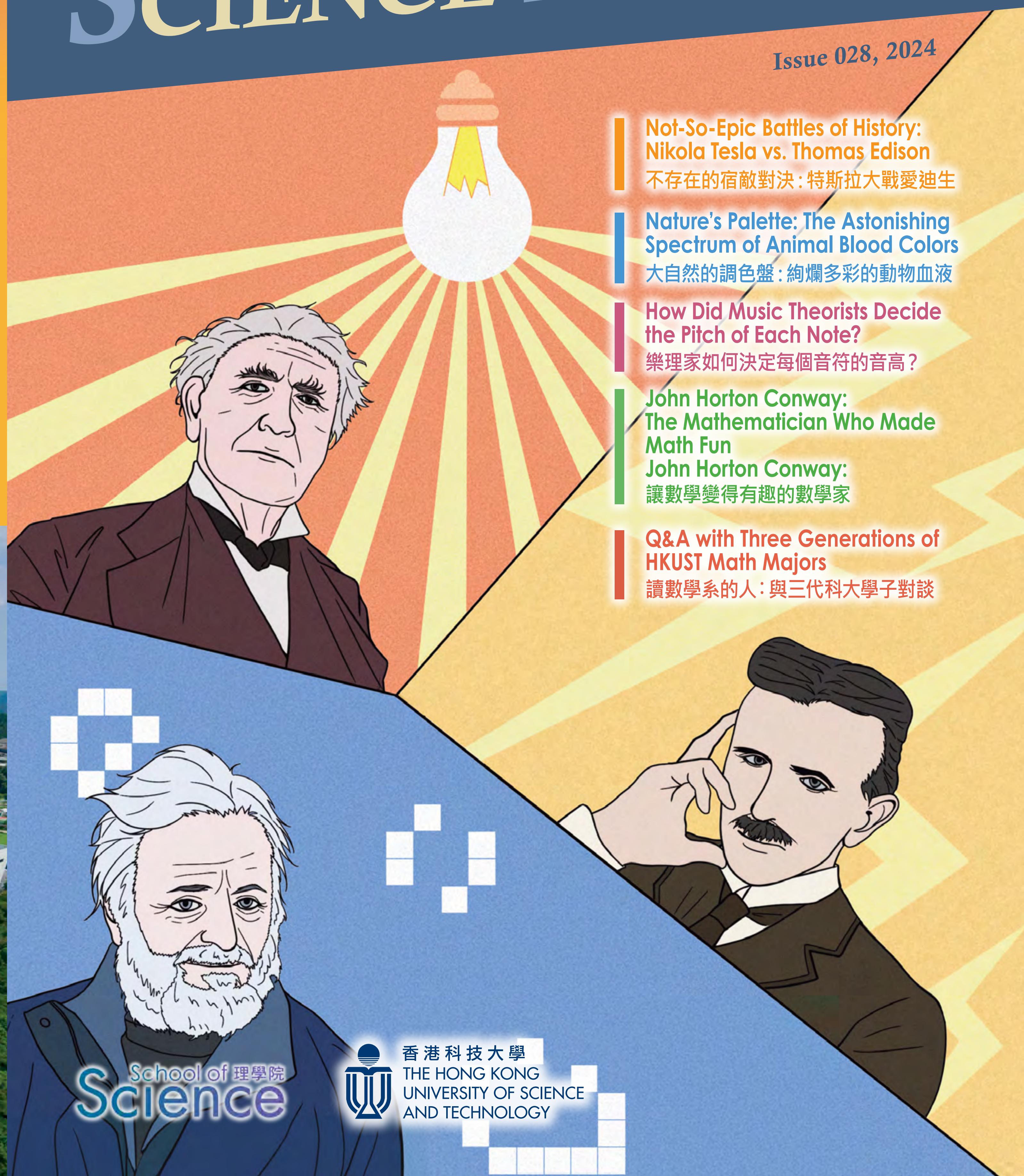
SCIENCE FOCUS 科言

Issue 028, 2024



School of 理學院 Science

香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Contents

Science Focus Issue 028, 2024

Message from the Editor-in-Chief 主編的話

Dear Readers,

Did you recognize the three people on the front cover? They are Thomas Edison, Nikola Tesla and John Conway. In this issue, you will learn not only about their scientific discoveries, but their personal stories. After all, we are all humans: Professional achievements can be linked to personalities, and all aspects of life. Along this line, you will find the first of a new series of articles when we discover personal and professional choices of HKUST students and graduates. First, we will hear from three generations of mathematicians, on their lives before, during and after studying at the HKUST. Last but not least, you will learn about how music and mathematics intertwine and the unexpected colors of animal blood.

At the start of a new academic year, let me wish you all the best inside and outside the classroom!

Yours faithfully,
Prof. Ho Yi Mak
Editor-in-Chief

親愛的讀者：

你認得出封面的三位人物嗎？他們是 Thomas Edison、Nikola Tesla 和 John Conway。今期我們不僅會探討他們對科學的貢獻，還會了解他們背後的故事。最終我們也是人：事業上的成就與個人性格，還有生命中的種種都息息相關。沿著這道脈絡，我們會向大家呈獻與科大學生和校友對談的首部曲，了解他們在個人和事業上的選擇。這次我們會先訪問「三代」讀數學的人，聽聽他們入讀科大前、中、後的心路歷程。最後，我們會探討音樂與數學密不可分的關係，以及出乎意料的血液顏色。

新學年伊始，在此祝各位校園內外的生活愉快！

主編 麥晧怡教授
敬上

What's Happening in Hong Kong? 香港科技活動

Fun in Fall Science Activities 秋日科學好節目

Any plans for this fall? Check out the following events!
計劃好這個秋天的好去處了嗎？不妨考慮以下活動！

Extinction · Resilience 滅絕 • 新生

The permanent exhibition "Extinction · Resilience" at the Science Museum explores the incredible journey of life on Earth over billions of years. Visitors are invited to learn more about prehistoric biodiversity, mass extinctions, and the remarkable resurgence of life by discovering around 80 sets of remarkable fossil specimens, including the three-billion-year-old stromatolites and the most complete *Deinonychus* fossil. The lifelike animatronic dinosaurs and engaging interactive exhibits will also amaze visitors by bringing the past to life. After the revamping of exhibits this April, a 423-million-year-old *Cooksonia barrandei* fossil, one of the earliest land plants on Earth, and 70-million-year-old elongated dinosaur eggs will also be on display.

科學館的常設展覽「滅絕 · 新生」將帶大家探索數十億年以來地球生命不可思議的演化旅程。透過約 80 組珍貴化石，包括擁有 30 億年歷史的疊層石和迄今最完整的恐爪龍化石，館方希望參觀者能藉此了解更多關於史前生物多樣性、大滅絕事件和地球生物絕處逢生的故事。展覽中栩栩如生的機械恐龍和有趣的互動展品亦能引領大家回到過去。在今年四月更換展品後，展覽增設四億 2300 萬年前最早陸生植物之一的頂囊蕨化石，以及 7000 萬年前的長形恐龍蛋。

Venue: G/F Palaeontology Gallery, Hong Kong Science Museum

地點：香港科學館地下古生物展廳

China's Lunar and Mars Exploration 中國登月探火

Explore the exhibition "China's Lunar and Mars Exploration" at the Space Museum, where the wonders of space come to life! This exhibition highlights China's significant achievements in lunar and Mars missions, showcasing full-scale models of the Yutu lunar rover and Zhurong Mars rover. Visitors may discover the evolution of China's space technology, from ancient poetic musings about the Moon to groundbreaking advancements in artificial satellites and manned spacecraft.

太空館的「中國登月探火」展覽使太空探索歷程上的奇蹟活現眼前！這個展覽介紹中國月球和火星探測任務上的偉大成就，當中展示「玉兔號」月球車和「祝融號」火星車的實物原大模型。從古人對月亮詩情畫意的聯想，到建造人造衛星和載人太空船上的突破，參觀人士都可以從展覽中了解中國航天發展的卓越成就。

Period: Now – March 24, 2025
Venue: Foyer, Hong Kong Space Museum
Admission fee: Free admission

展期： 現在至 2025 年 3 月 24 日
地點： 香港太空館大堂
入場費： 免費入場

Not-So-Epic Battles of History:

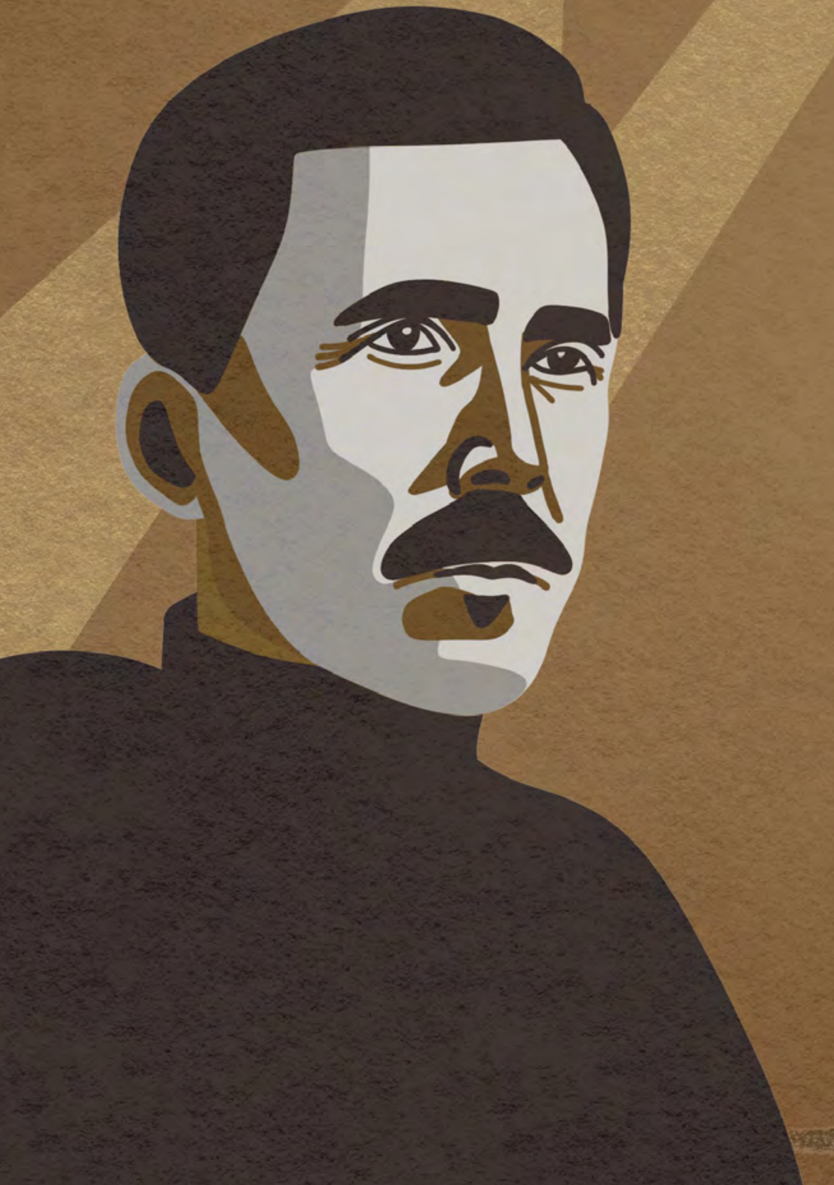
不存在的宿敵對決：

Nikola Tesla VS. Thomas Edison

特斯拉大戰愛迪生

By Aastha Shreeharsh

There are many colorful characters, discoveries, and inventions in the annals of scientific history; yet, not much has sparked as much debate and controversy as the infamous rivalry between Nikola Tesla and Thomas Edison. Ranging from episodes in the popular sitcom The *Big Bang Theory*, the witty illustrations in the comic *The Oatmeal*, and YouTube videos from channels such as "Epic Rap Battles of History", the Tesla vs. Edison narrative has become a persistent fixture in popular culture. As reflected in such media, Tesla is hailed by many as the unrecognized genius of the duo, the protégé that surpassed his mentor, Edison. Meanwhile, Edison, who was once hailed by his contemporaries, the media and even our history textbooks as one of the greatest inventors of all time, is now considered a hack by many. Unfortunately for avid Tesla fans (and fortunately for fans of Edison), reality seems to be more nuanced than this trumped-up rivalry.



In 1884, the Serbian physicist Nikola Tesla, aged 28, arrived in New York City and took up employment with Edison [1]. Nine years his elder, Edison had already established himself as a success with his invention of a new kind of telegraph, and established the Edison Illuminating Company which furthered his own work in electric light. Tesla, in his recently acquired position, assisted Edison in installing equipment, repairing generators, and designing new machines. Edison's work relied on the principles of direct current (d.c.), which was the national standard at that time, and Edison was profiting off many patents (Footnote 1) for his electric lighting system that utilized d.c. [1, 2]. However, Tesla saw promise in utilizing alternating current (a.c.), so a year later, he quitted working for Edison and set up his own electric company – one that utilized a.c. – thus setting the scene for the Tesla vs. Edison narrative.

Current is the flow of "electricity," or more accurately, the flow of charged particles like electrons and positive ions. As the name suggests, d.c. flows in one direction while a.c. alternates its direction back and forth in a single second. So, why did Edison and Tesla prefer using different systems of current?

For electricity to be transmitted over long distances in a city, the major challenge is to overcome power loss. To minimize power loss, which can be represented by the formula $P = I^2R$ (where P, I and R denote power loss, current and resistance of the cable respectively), electric energy should be converted to a high-voltage, low-current form before transmission. While d.c. was widely used in the US at the time, it was challenging to convert low-power plant voltages into higher voltages, necessitating many small power plants near users. Tesla's a.c. system resolved this challenge by harnessing the nature of a.c.; a.c. voltage can be easily stepped up and down by a transformer through electromagnetic induction, due to the alternating input current and the induced change in magnetic field. The high efficiency of transmission enables consumers to utilize electricity from power plants miles away [1]. As the party with vested interest, Edison seemed to have motive for waging a smear campaign, play dirty and discredit poor Tesla's ideas, just as the prevailing narrative suggests.

The infamous "current wars" did take place; however, the rivalry between Tesla and Edison was not nearly as personal and epic as people would like to believe. First and foremost, these "current wars" took place after Tesla's patents were acquired by George Westinghouse [1]. Thus, it was Westinghouse who promoted the a.c. system against Edison, not so much Tesla. Secondly, both Edison and Westinghouse did attempt to discredit each other's system and promote their own, but Edison lost within a matter of a few years. In 1893, Westinghouse secured the World's Fair electrification bid [1]. By 1896, General Electric, a leading company co-founded by Edison, switched from d.c. to a.c., paving the way for a.c. as the dominant system in the US [1]. Concurrently, Tesla was swiftly moving onto new inventions [1]. Essentially, the "current wars" were almost entirely a commercial dispute.

Then, how did the "Tesla good, Edison bad" narrative prevail? This may be related to Edison's shock tactics to discredit Tesla's ideas. To justify the argument that a.c. is more dangerous than d.c., Edison's West Orange Laboratory has conducted research on electrocution [3, 4], in which various "unwanted" animals like dogs, calves, and a horse were killed [4]. In 1903, despite having essentially lost the a.c. vs. d.c. battle in the US, Edison was associated with the public display of the electrocution of the elephant, Topsy [1, 3, 4], who was sentenced to death after the circus elephant had killed a man and been proved unmanageable (but actually Topsy was physically abused before attacking people in those episodes [3, 4]). As a failing attempt at relevance (both a.c. and d.c. can be dangerous!) and distressing act of animal cruelty, this incident probably does not paint Edison in the best light by today's standards, although Edison's supporters would argue that Edison's research provided a more humane way for Topsy's execution comparing to hanging, and that Edison might not be personally involved in this incident [3, 4].

The working style of Edison and Tesla varies, too. Famous for his expertise in patenting ideas and dealing with the press, Edison was a businessman through and through. He was also considered a pure empiricist who achieved breakthroughs through endless trials and experiments, while Tesla might favor a different approach as a theorist [5]. We should add that Tesla was not as terrible a businessman as modern-day geeks make him out to be. Being not as commercially minded as Edison, he conceived utopian ideas, such as wireless transmission, with a long-term vision of what technology can do [1].

In today's capitalistic culture, the Silicon Valley ideals of ingenuity, efficiency, and novel ideas are what make the world go round. Intellectuals proudly wearing the badge of "nerd" in shows like *The Big Bang Theory*, bemoaning comparisons to Edison and rejoicing in those to Tesla, are reflective of a shift in modern values. However, let us recognize both scientists' contributions that have and will impact the past, present and future of science. In many cases, truth is not as dramatic as people want to believe. The reality of the Tesla vs. Edison showdown reminds us

that fame is fleeting and so is our recollection of history – which is why, it is all the more important that we do not hastily idolize or demonize historic figures. After all, they were human too.

1. Patent: A legal document that grants the inventor of a new invention the exclusive right to make, use, and sell that invention for a certain period of time, usually 20 years from the date of filing the application. Throughout Edison's life, he acquired an astonishing number of 1,093 patents [2].

科學史冊上記載著各種多姿多彩的偉人、發現和發明，然而沒有甚麼像尼古拉・特斯拉（Nikola Tesla）和湯馬士・愛迪生（Thomas Edison）這對知名宿敵之間的較量一樣能引發激烈的辯論和爭議。從流行處境喜劇《囧男大爆炸》（The Big Bang Theory），到漫畫《The Oatmeal》中的詼諧描繪，以及經典饒舌爭霸戰（Epic Rap Battles of History）等 YouTube 頻道的影片，「特斯拉大戰愛迪生」的故事已成為流行文化中的一個經典戲碼。正如這些媒體所反映的那樣，許多人將特斯拉譽為寂寂無名的天才，相信他更勝其師父愛迪生；與此同時，曾被同時代的人、媒體，甚至我們歷史教科書稱為有史以來最偉大發明家之一的愛迪生，現在卻被許多人蔑視為一個過譽的庸才。雖說如此，特斯拉的忠實粉絲未必想聽到的是（或是對愛迪生粉絲來說值得高興的是），現實中二人關係似乎比傳聞中虛構的對立來得複雜。

1884 年，當時 28 歲的塞爾維亞物理學家尼古拉・特斯拉來到紐約，受聘於愛迪生的公司 [1]。愛迪生比他大九歲，當時已因發明了一種新型電報取得成功，又創立了愛迪生照明公司進一步改良自己發明的電燈。特斯拉則在剛入職的職位中協助愛迪生安裝設備、修理發電機和設計新機器。愛迪生的系統採用當時的國家標準，依靠直流電運作，而愛迪生本身亦透過許多直流電照明系統的專利權（註一）獲取利潤 [1, 2]。然而，特斯拉看到利用交流電的潛力，因此在一年後辭去工作，成立了使用交流電的電力公司，亦為「特斯拉大戰愛迪生」的故事寫上序幕。

電流是「電」的流動，更準確地說，是電子或正離子等帶電粒子的流動。正如字面上的意思，直流電是單向流動的，而交流電則會在一秒內不斷改變方向。那麼，愛迪生和特斯拉為甚麼喜歡使用不同的電流系統呢？

在城市中長距離傳輸電力的主要挑戰是克服功率損耗。功率損耗可以用公式 $P = I^2R$ 表示（當中 P, I 和 R 分別為功率損耗、電流和電纜的電阻），因此電能應先被轉換成高電壓、低電流的形態來進行傳輸以減少功率損耗。雖然直流電在當時的美國被廣泛使用，但要從發電廠的低電壓轉為較高的電壓非常具挑戰性，需要在靠近用戶一端興建大量小型發電站。特斯拉的交流電系統正正利用交流電的特性解決了這個難題：不斷改變的電流方向和誘導磁場使電壓能在變壓器中透過電磁感應的原理輕鬆升降，帶來的高效傳輸使用戶可在距離發電廠數以哩計的家中享用電力 [1]。恰如主流論調所認為的一樣，愛迪生作為既得利益者，似乎就有了合理動機發起一輪誹謗攻勢，以卑鄙手段抹黑特斯拉的系統。

著名的「電流大戰」確實就這樣發生，但特斯拉與愛迪生之間的競爭並沒有人們想像的那麼史詩式，二人也沒有在個人層面上交鋒。首先，「電流大戰」發生於特斯拉的專利被喬治・威斯汀豪斯（George Westinghouse）收購之後 [1]，因此推廣交流電系統以抗衡愛迪生的是威斯汀豪斯，而不是特斯拉。其次，無論是愛迪生或威斯汀豪斯都曾試圖抹黑對方的系統並宣傳自己的系統，但愛迪生在僅僅幾年就宣告敗北。1893 年，威斯汀豪斯投得為世界博覽會提供電力的標書 [1]；到 1896 年，由愛迪生共同創立的領頭公司通用電氣改用交流電，為交流電成為美國主要電力系統埋下伏線 [1]。同時，特斯拉很快就改為研究其他新發明了 [1]。「電流大戰」基本上只是一場商業糾紛。

那麼「特斯拉是好人，愛迪生是反派」的想法是如何興起的呢？這可能與愛迪生抹黑特斯拉系統的招數有關。為了證明交流電比直流電更危險，愛迪生的西奧蘭治實驗室（West Orange Laboratory）進行了關於電刑的研究 [3, 4]，過程中處決了各種「沒有人想收養」的動物，例如狗、小牛和一匹馬 [4]。儘管「交流電對直流電」一戰於 1903 年在美國基本上大局已定，但愛迪生卻在此時與公開處決馬戲團大象 Topsy 一事扯上關係 [1, 3, 4]，當中大象因殺害一名男子，以及被馬戲團認定為無法控制，因而被判處極刑（但其實 Topsy 每次攻擊人類前均遭受虐待 [3, 4]）。這次殘酷對待動物的行為不但未能證明交流電比較危險（事實上兩者也可以很危險！），更令人痛心疾首，以現今標準來看這件事可謂將愛迪生的形象毀於一旦，儘管其支持者會辯稱愛迪生的研究為 Topsy 提供了比絞刑更人道的處決方式，而愛迪生亦可能沒有親身參與事件 [3, 4]。

愛迪生和特斯拉在工作風格上也有所不同。愛迪生以擅於將想法轉化成專利權和應對傳媒的手腕聞名，是個不折不扣的生意人。他也是一位純經驗主義者，靠無止境地不斷嘗試和進行實驗來尋求突破，反之特斯拉作為理論主義者很可能會採取截然不同的策略 [5]。另一方面，雖然特斯拉作為商人並不像現代科技宅所說的那麼糟糕，但他也沒有像愛迪生那樣側重商業考慮，因此能構想出像無線傳輸這些具烏托邦色彩的概念，以獨特的遠見向世人展現科技在未來的各種可能性 [1]。

在現今的資本主義文化中，矽谷所追求的足智多謀、高效和嶄新概念才是推動世界運轉的原動力。從《囧男大爆炸》等節目中可以看到現今知識分子對「書呆子」這些本為貶意的稱號毫不在乎；他們一方面抗拒人們拿愛迪生和自己比較，另一方面渴望擁有特斯拉的特質，這些都反映現代價值觀的轉變。然而，我們都不能否認這兩位科學家的貢獻對科學都有著深遠的影響，除了改變科學的過去和現在，影響力還會持續至未來。很多時候，現實並不像人們所想的那樣戲劇化。特斯拉與愛迪生對決的真相提醒了我們名利是短暫的，人們對歷史的記憶也是如此 — 這就是為什麼我們不應盲目地將歷史人物神話化或妖魔化，而他們終究也是人。

1 專利：授予發明者獨家權利在一定時間內（通常從提交申請一天起計的 20 年）製造、使用和銷售該發明的法律文件。愛迪生一生中擁有過高達 1,093 項專利 [2]。

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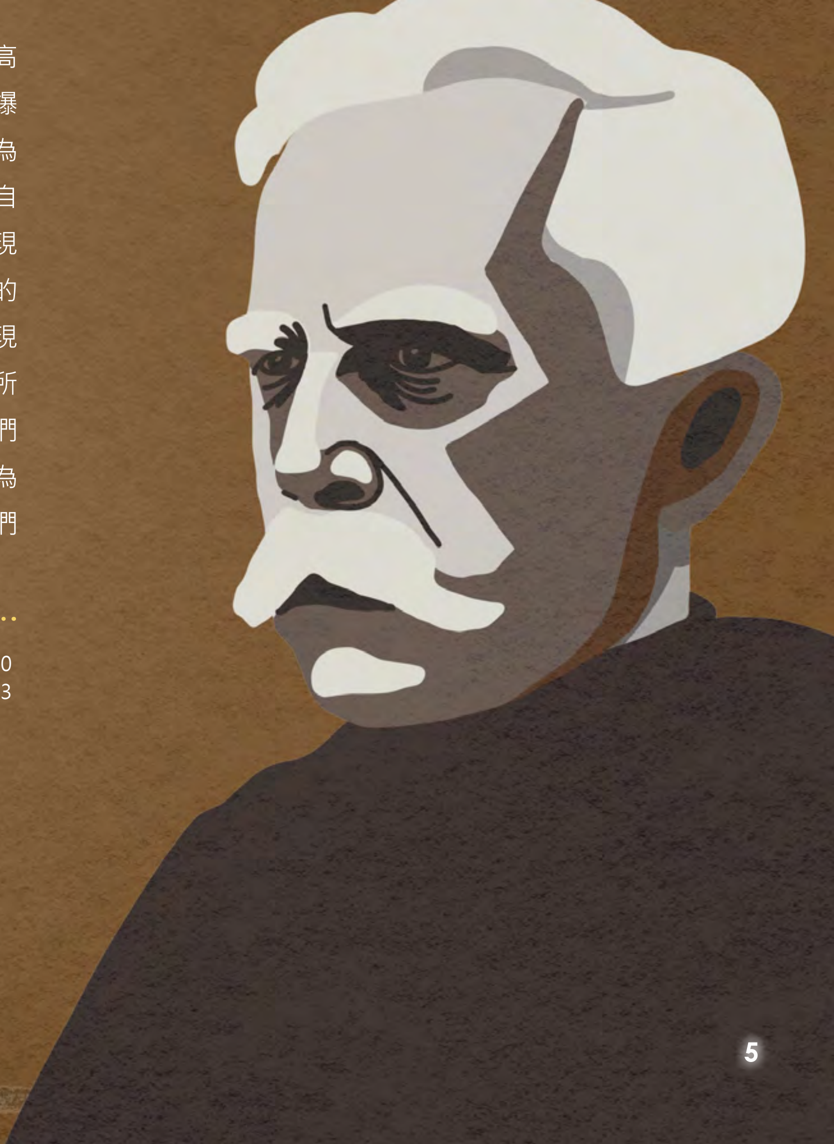


Nature's Palette 大自然的調色盤：The Astonishing Spectrum of Animal Blood Colors 絢爛多彩的動物血液

By Helen Wong 王思齊

What is the color of blood?

While the instinctive response for many of us may be "red," the truth is that nature's artistry extends far beyond our imagination. Let's delve into the astonishing world of blood's vibrant hues – where blues, purples, greens, and even the absence of color thrive [1, 2].

Determinant of Blood Color: Respiratory Pigment

The key to this breathtaking diversity of animal blood colors lies in the type(s) of respiratory pigment in our blood. Respiratory pigments are metal-containing proteins that help transport respiratory gasses such as oxygen and carbon dioxide (footnote 1). The well-known hemoglobin in our blood is just one example of such a pigment, along with lesser-known ones like chlorocruorin, hemocyanin, and hemerythrin. These pigments differ in their chemical structures and use different metal complexes to bind oxygen molecules. Together, these subtle variations cause each pigment to absorb and reflect unique wavelengths of light, resulting in a vast array of blood colors.

Be careful: When we talk about blood color, we are referring to the color of the oxygenated pigments rather than the deoxygenated ones. When oxygen binds to the metals within these pigments, it alters the three-dimensional structures of the whole pigments and, in some cases, changes the oxidation states of the metals [2]. This, in turn, results in a shift in their light absorption and reflection spectra, changing the blood colors we see.

Blue Blood

In certain invertebrates such as squid, octopus, lobster, and horseshoe crab, the presence of the respiratory pigment hemocyanin gives their blood a distinct blue color [2]. Unlike hemoglobin, which utilizes iron (Fe^{2+}) to bind oxygen, hemocyanin relies on copper (Cu^{2+}) for oxygen transportation in these marine animals. The copper (II) ion strongly absorbs red light while reflecting blue light, resulting in the characteristic blue appearance of their blood.

These blue-blooded invertebrates have evolved to use hemocyanin for two reasons [3]. Firstly, the effectiveness of hemoglobin to transport oxygen decreases at low temperatures, such that hemocyanin outperforms hemoglobin in the deep sea. Secondly, hemoglobin exhibits superior efficiency in binding oxygen compared to hemocyanin but only in oxygen-rich environments, because the binding of every new oxygen molecule can facilitate that of the next oxygen molecule, until the four vacancies in the hemoglobin are taken up. However, in oxygen-deprived settings, hemoglobin's oxygen-binding efficiency diminishes, and hemocyanin proves to be more effective. Hence, the switch to hemocyanin gives these marine animals an edge in obtaining oxygen in the ocean.

Purple Blood

Blood naturally takes on a purple color in lamp shells and certain marine worms [1, 2]. These marine invertebrates use neither hemoglobin nor hemocyanin, but hemerythrin. While hemerythrin, like hemoglobin, uses iron as the oxygen-binding material, it gives a violet-pink color instead of bright red in its oxygenated state, and is colorless when no oxygen is bound.

Green Blood

For some earthworms and leeches, green blood is the norm [1, 2]. These invertebrates contain chlorocruorin, another iron-based pigment that makes their blood look green. While chlorocruorin is typically linked to green hues, its color is concentration-dependent – lower amounts appear green, but higher concentrations cause the pigment to take on red coloration.

At the molecular level, chlorocruorin closely resembles hemoglobin. In fact, its chemical composition diverges from hemoglobin in just one respect: Chlorocruorin contains an aldehyde group ($-CHO$) whereas hemoglobin has a vinyl group ($-CH=CH_2$). Nevertheless, it is worth noting that chlorocruorin does not contain chlorine as its name may otherwise suggest.

Here comes an interesting twist: Green blood does not solely rely on the presence of chlorocruorin. Like most vertebrates, green-blooded skinks from New Guinea use hemoglobin to carry oxygen. However, their blood and tissues are green [4]. This peculiar phenomenon is related to how these lizards recycle hemoglobin. In humans, the recycling of hemoglobin involves two steps, first by breaking the pigment down into a green chemical named biliverdin, then by converting biliverdin to a yellow compound called bilirubin. However, the lizards lack the ability to further metabolize biliverdin, leading to an accumulation of the green pigment in their blood [1, 2]. The color is so intense that it overshadows the natural red color of hemoglobin.

Colorless Blood

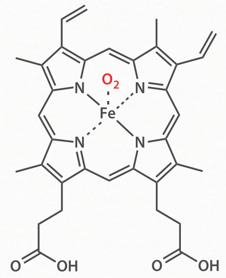
Perhaps the strangest "blood color" is having no color at all. The Antarctic icefishes stand out as one of the most unusual vertebrates as they lack any respiratory pigment [2, 3]. As a result, icefish blood is colorless with only blood plasma. Despite the absence of hemoglobin, scientists have discovered remnants of hemoglobin genes in icefish genomes, suggesting that these genes were lost during the course of evolution [5].

But the question remains: How do these Antarctic fishes survive without such an important oxygen carrier? Icefishes have developed several adaptations to compensate for that [2]. They have a larger blood volume than related fish species, and lead a relatively sedentary lifestyle, which helps reduce their oxygen demands. On the other hand, the cold water of the Southern Atlantic also promote icefish survival by maintaining higher concentrations of dissolved oxygen than warmer seas.

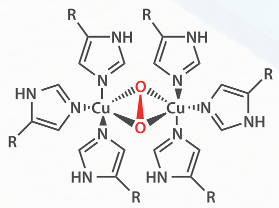
Conclusion

Now back to the same question: What is the color of blood?

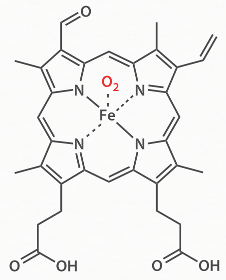
This time, you might be tempted to respond with a range of colors we have covered – red, blue, purple, green, and even colorless – and they are all right! For other creative answers that you may think of, why not? Scientific knowledge is tentative and subject to change when new evidence appears – so who knows, perhaps there are yet undiscovered ones waiting to join the ever-growing palette of blood colors!



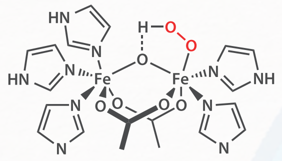
Hemoglobin (Heme B)



Hemocyanin



Chlorocruorin



Hemerythrin

1. Editor's note: While most of the carbon dioxide is transported through blood plasma in the form of bicarbonate ions in humans (as mentioned in high school textbooks), 10% of the gas is actually carried by hemoglobin [6].

血是甚麼顏色的呢？

大多數人可能會首先想到紅色，但事實上，大自然的創造力遠超我們想像。讓我們一起探索血液色彩的奇妙世界 — 那裡有藍色、紫色、綠色，甚至是透明 [1, 2]！

決定血液顏色的因素：呼吸色素

血液顏色之所以擁有如此驚人的多樣性，關鍵在於動物血液中呼吸色素的種類。呼吸色素是一種含有金屬的蛋白質，作用是運送氧和二氧化碳等呼吸氣體（註一）。我們熟悉的血紅蛋白只是其中一種，其他較少人熟悉的包括血綠蛋白、血藍蛋白和蚯蚓血紅蛋白。這些呼吸色素各有不同的化學結構，並使用不同的金屬絡合物與氧分子結合。正是這些微妙的差異使每種色素吸收和反射獨特波長的光線，從而產生各種各樣的血液顏色。

值得留意的是，在討論血液顏色時，我們指的是含氧色素而非缺氧色素的顏色。當氧與呼吸色素中的金屬結合時，會改變色素的三維結構，在某些情況下金屬的氧化態亦會產生變化 [2]。這會改變色素的吸收和反射光譜，進而改變我們所看到的血液顏色。

藍色血液

在某些無脊椎動物中，如魷魚、章魚、龍蝦和鱟，血藍蛋白的存在使牠們的血液呈現明顯的藍色 [2]。與血紅蛋白利用鐵（Fe^{2+}）與氧結合不同，這些海洋生物中的血藍蛋白依靠銅（Cu^{2+}）來運送氧。銅（II）離子強烈吸收紅光並反射藍光，使這些海洋動物的血液呈現獨特的藍色。

進化最終使這些無脊椎動物使用血藍蛋白的原因有兩個 [3]。首先，血紅蛋白在低溫下運送氧的效率會下降，因此血藍蛋白在深海的表現比血紅蛋白更佳。另外，血紅蛋白僅在富氧環境下與氧結合的能力較強，因為每個氧分子與血紅蛋白結合後都會促進下一個分子的結合，直至四個位置被佔滿為止；然而在缺氧環境中，血紅蛋白與氧的結合在缺乏這種加乘下，效率將大幅降低，使血藍蛋白反而更勝一籌。因此，使用血藍蛋白能使這些海洋生物在獲取氧方面取得優勢。

紫色血液

腕足動物和一些海洋蠕蟲的血液呈現紫色 [1, 2]。這些海洋無脊椎動物均沒有血紅蛋白或血藍蛋白，而是含有蚯蚓血紅蛋白。儘管蚯蚓血紅蛋白和血紅蛋白都使用鐵作為氧結合物質，但前者在含氧狀態下呈現粉紫色而非鮮紅色，在缺氧狀態下則不呈現任何顏色。

綠色血液

對某些蚯蚓和水蛭來說，綠色的血再正常不過 [1, 2]。這些無脊椎動物含有血綠蛋白，一種使血液呈綠色的鐵基色素。雖然血綠蛋白通常被認為是綠色，但其顏色其實取決於濃度：較低含量時呈現綠色，但較高濃度時呈現紅色。

從分子層面來看，血綠蛋白與血紅蛋白非常相似。事實上，其化學結構僅在一個位置與血紅蛋白不同：血綠蛋白在那個位置含有醛基（-CHO），而血紅蛋白則含有乙烯基（$-CH=CH_2$）。然而值得注意的是，儘管血綠蛋白的英文名稱「chlorocruorin」以「chloro」開首，但它並不像其含義一樣含有氯（chlorine）。

然而事情並不就此作結：綠色血液有時與血綠蛋白無關。和大多數脊椎動物一樣，新幾內亞的綠血石龍子（屬於石龍子科的蜥蜴）以血紅蛋白運送氧，但牠們的血液和組織卻呈綠色 [4]。這個奇特的現象與這些蜥蜴如何回收血紅蛋白有關。人類中血紅蛋白的回收涉及兩個步驟：首先是將血紅蛋白分解成一種稱為膽綠素的綠色化學物質，然後是將膽綠素轉化為另一種名為膽紅素的黃色化合物。然而，這些蜥蜴缺乏代謝膽綠素的能力，導致該色素在血液中積聚 [1, 2]。由於膽綠素的顏色過於強烈，以至掩蓋了血紅蛋白的天然紅色。

透明血液

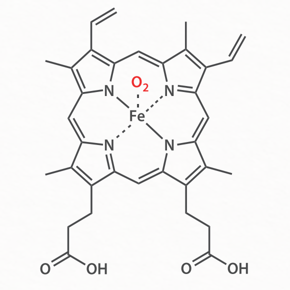
也許最奇怪的「血色」是沒有顏色。南極冰魚是最不尋常的脊椎動物之一，因為牠們沒有任何呼吸色素 [2, 3]，因此只有血漿的冰魚血液是無色的。儘管現代冰魚沒有血紅蛋白，但科學家在牠們的基因組中還是發現了血紅蛋白基因的殘骸，意味著血紅蛋白基因可能是在進化過程中丟失 [5]。

這衍生出一個耐人尋味的問題：沒有了如此重要的氧載體，南極冰魚到底是如何生存的呢？原來冰魚為此發展出多種適應機制 [2]：它們擁有比近親魚類更大的血液體積，並通過相對靜態的生活模式來減少自身的氧需求。另一方面，南大西洋的寒冷水域相比其他溫暖海域有更高的溶氧度，這亦有利於冰魚生存。

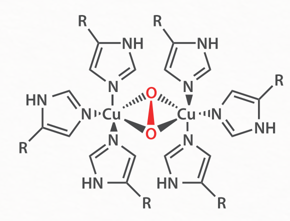
結論

現在回到最初的問題：血是甚麼顏色的呢？

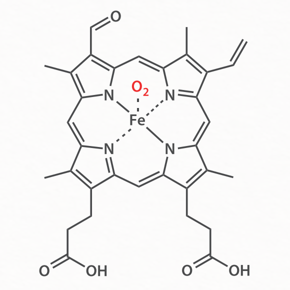
這次，你可能會給出我們所討論過的一系列顏色 — 紅色、藍色、紫色、綠色，甚至是透明 — 它們全都是正確的！至於其他你可能想到極具創意的答案，又有何不可呢？科學知識是暫時性的，意味著當有新證據出現時，我們的認知亦可能隨之改變 — 說不定還有一些未被發現的顏色正等待加入這個燦爛多彩的血液大家庭呢！



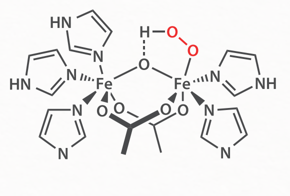
血紅蛋白（血基質 B）



血藍蛋白



血綠蛋白



蚯蚓血紅蛋白

1 編按：雖然大部分二氧化碳在人體中都以碳酸氫鹽離子的形式透過血漿運輸（正如高中教科書所提到的一樣），但實際上 10% 的二氧化碳是由血紅蛋白攜帶的 [6]。

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How Did Music Theorists Decide the Pitch of Each Note?

樂理家如何決定每個音符的音高？

By Jane Yang 楊靜悠

Introduction

Known as the temperament or scale in music theory, we learn to sing "do-re-mi-fa-sol-la-ti" as early as kindergarten. We also learn that all sounds are essentially produced by the vibrations that hit our eardrum, whose frequency decides the pitch. Then, have you ever thought about how music theorists chose a pitch for each note in "do-re-mi-fa-sol-la-ti" from an infinite number of options on a number line? In this article, we will introduce you to "Pythagorean temperament," an early musical scale often attributed to an ancient Greek mathematician, Pythagoras [1, 2]. We will also delve into how music theorists and mathematicians later developed "equal temperament," which has become the most widely used musical scale in western music since the 19th century [1].

Pythagorean Temperament

First of all, let's understand the concept of an "octave." Mathematically, two sounds are considered an octave apart if their frequencies have a ratio of 2:1. For example, the standardized "middle A" has a frequency of 440 Hz (Footnote 1), or vibrations per second, while the next A which is an octave higher has a frequency of 880 Hz. When they are played at the same time, they sound so consonant that the human brain perceives them as the "same" note but the latter in a higher pitch. This phenomenon is called "octave equivalence [1, 2]."

Therefore, to create a musical scale, we only need to consider an octave, or one cycle of "do-re-mi-fa-sol-la-ti". We can then multiply or divide the frequencies of the notes in an octave by any power of two to obtain a higher or lower octave because of octave equivalence. Pythagoras also discovered two notes that are a "fifth" apart, meaning their frequencies have a ratio of 3:2, also sound pleasant when played together. Hence, he decided that the task was to create as many ratios of 3:2 and 2:1 as possible to provide convenience for composers.

Obviously, Pythagoras should have no access to the accurate frequency of each note, so the tuning was probably completed by hearing the pitch and comparing its relative distance to the base note. However, for a better understanding, let's unveil the ancient method based on our modern understanding.

To decide a frequency to each of the notes in an octave, Pythagoras started with the note A at 440 Hz and multiplied its frequency by 3/2 to obtain the note at 660 Hz. By multiplying 3/2 again, he got 990 Hz. However, this exceeded the desired octave range (i.e. greater than 880 Hz), so he divided it by two to get the note equivalent to it at 495 Hz. Pythagoras repeated this process of multiplying by 3/2, and dividing by two if the resulting frequency exceeds 880Hz, until he obtained a musical scale consisting of seven nonequivalent notes which is enough to play simple melodies [1]. He rearranged those frequencies in order, creating a musical temperament very similar to the one we use today (Table 1).

Equal Temperament

However, the seven notes in Pythagorean temperament are just enough for playing simple melodies. Before we examine the problem of Pythagorean temperament, let's look at the modern system called "equal temperament". This temperament divides an octave into 12 equal musical intervals. Keep in mind that our brain perceives the distance of musical interval by ratio instead of difference. Therefore, the frequencies of each note in a scale should have an exponential relation, with a ratio r between each pair of adjacent notes satisfying $r^{12} = 2$, i.e. $r = 2^{1/12}$. By multiplying the starting frequency by the ratio $r = 2^{1/12}$ for 12 times, we obtain the frequencies of all the notes within an octave (Table 2).

Key Change

So, why is equal temperament preferred over Pythagorean temperament? You may have heard of a musical jargon called "key change" before. Actually, the mathematical implication of key change is to multiply the frequency of each note of a melody by a constant number. After performing this trick, human brains will still perceive the two melodies as the same since the musical interval (i.e. the frequency ratios) between any two adjacent notes are retained [1]. For example, a melody that plays 440Hz, 660Hz, and 733.3Hz in order is considered equivalent to a melody that plays 550Hz, 825Hz and 916.6Hz. Key change in music usually helps musicians express their feelings: Changing to a higher key in the midway of a piece of music can express excitement or encouragement,

Ratio	1	9/8	81/64	4/3	3/2	27/16	243/128	2
Frequencies(Hz)	440	495	557	587	660	743	835	880

Table 1 The frequencies of notes and their ratios with respect to the note A in Pythagorean temperament. The values are rounded off to the nearest integer.

Ratio	1	$2^{1/12}$	$2^{2/12}$	$2^{3/12}$	$2^{4/12}$	$2^{5/12}$	$2^{6/12}$
Frequencies(Hz)	440	466	494	523	554	587	622
Ratio	$2^{7/12}$	$2^{8/12}$	$2^{9/12}$	$2^{10/12}$	$2^{11/12}$	2	
Frequencies(Hz)	659	698	740	784	831	880	

Table 2 The frequencies of notes and their ratios with respect to the note A in equal temperament. The values are rounded off to the nearest integer. The frequencies in shaded cells are played by the black keys of a piano.

while lowering the key may convey sorrow or tranquility. In addition, by lowering the key of a song, a singer whose voice range is too low to cover the high pitch can now sing the song.

After understanding the concept of key change, you would discover that the equal temperament adapts to key change perfectly because the ratio of the frequencies between any adjacent notes is a constant [1, 2]. Pianists, for example, only need to move up every note for one key on a piano keyboard tuned with equal temperament to complete the key change, and the finite number of keys on the keyboard is sufficient to cover all notes required for any key changes.

On the other hand, the seven notes in Pythagorean temperament don't suffice. Instead of having a constant ratio, adjacent notes in Pythagorean temperament have a ratio of either 9:8 or 256:243 [2]. We have to continue Pythagoras' calculation to create more and more notes so that key changes can be performed perfectly from any note. By extending his calculation beyond the first octave, we wish the value will return to the starting point 440 Hz at some point, so that we can get a finite number of notes. Nevertheless, this has been proved impossible, due to the fact that $(3/2)^n$ is never a power of two, so we will need an infinite number of black keys for a musical instrument to perform key changes, which is simply not practical [1]. Although Pythagoras was able to get close to the desired frequency 440 Hz, there was still a small discrepancy known as the "Pythagorean comma" [2]. This slightly higher frequency ratio of 1.0136:1 posed challenges for musicians and mathematicians until the invention of equal temperament (Footnote 2).

Historical Controversies Over the Invention of Equal Temperament

One interesting coincidence is that the equal temperament was invented by the Chinese mathematician, physicist and music theorist Zhu Zaiyu in 1584, and given a mathematical definition by the Flemish mathematician Simon Stevin around the period between 1585–1608 [3]. There are still controversies on who should receive the credit and whether the development was independent [3, 4], but we may never know the truth.

Nevertheless, one thing you can take away is that anything we take for granted today may have been the outcome of the struggle of our predecessors for thousands of years, and there may actually be a scientific reason behind it. From Pythagoras' exploration to the invention of equal temperament, these mathematicians have shaped the music we enjoy. So next time you sing "do-re-mi-fa-sol-la-ti", remember the mathematical journey that led to these familiar notes.

1. Middle A: In the case of C major (one of the easiest modes in music), "do-re-mi-fa-sol-la-ti" corresponds to C, D, E, F, G, A, B respectively in representation. Chosen as a standard note for tuning musical instrument, the "middle A" corresponds to "la" in C major. Although it should have a frequency of 440 Hz by the ISO 16 standard [5], the tune is sometimes set at 442 Hz in some wind bands to cater the wind instruments.
2. Editor's note: The number 1.0136 is given by $3^{12} / 2^{19}$, i.e. taking the fifths 12 times while reducing the octaves seven times.

引言

我們從幼稚園時期就學會唱「do-re-mi-fa-sol-la-ti」，而在樂理中，這個概念被稱為音律或音階。我們還學過所有聲音本質上都是衝擊耳膜的振動，這些振動的頻率決定了音高。那麼，你有否思考過樂理家是如何從數軸上無窮多的選擇中，為「do-re-mi-fa-sol-la-ti」裡的每個音符選定音高呢？在這篇文章裡，我們將介紹「畢氏音律」— 一種普遍認定為由古希臘數學家畢達哥拉斯提出的早期音樂音階 [1, 2]。我們還會探討樂理家和數學家後來如何發展出「十二平均律」，這種音階自 19 世紀以來一直都是西方音樂最廣泛使用的音階 [1]。

畢氏音律

首先讓我們了解「八度」這個概念：在數學上，如果兩個音符的頻率比為 2:1，它們就是相差一個八度，例如標準「中音 A」的頻率為 440 Hz（註一），亦即每秒振動 440 次，那麼比它高一個八度的 A 的頻率則為 880 Hz。當同時演奏這兩個音時，重疊的聲音聽起來會和諧得使人腦認為它們是「相同」的音符，只是後者的音高較高而已，這種相似性被稱為「等價八度」[1, 2]。

因此，就創造一個音階而言，我們只需要考慮一個八度，亦即是循環裡的一次「do-re-mi-fa-sol-la-ti」。然後我們可以藉等價八度的特性，將八度內的音符頻率乘或除以任何二的次方數，以獲取更高或更低的八度。畢達哥拉斯還發現當兩個音符的頻率比為 3:2，即是相隔一個「五度」時，同時演奏這兩個音也會有非常悅耳的效果。因此，他決定音階裡要儘可能包含最多的 3:2 和 2:1 比例，以便作曲家創作。

顯然，畢達哥拉斯當時應該無法得知每個音符的準確頻率，因此調音可能只是透過聆聽音高來估計一個音與基準音之間的相對距離來完成。然而，為了方便理解，讓我們從現代角度揭示這古老的調律方法。

為了決定八度內每個音的頻率，畢達哥拉斯先從 440 Hz 的 A 音入手，將其頻率乘以 3/2 以獲得 660 Hz 這個音。透過再乘以 3/2，他得到 990 Hz，但這超出了八度範圍（即大於 880 Hz），因此他將其除以二以獲得等價的 495 Hz。畢達哥拉斯重複這個乘以 3/2，然後如果得出頻率超過 880 Hz 則把其除以 2 的過程，直至得到一個由七個不等價音符組成的音階為止，這音階足以演奏簡單旋律 [1]。他把這些音按頻率順序排列，創造出一個與現代版本非常相似的音律（表一）。

十二平均律

然而，畢氏音律中的七個音符僅僅足以演奏簡單旋律。在檢視畢氏音律的不足之前，讓我們先了解現代的「十二平均律」。這種音律將一個八度分為 12 個相等的音程。但要注意的是，我們大腦是以頻率間的比例而非差異來判斷兩個音之間的距離，因此音階中每個音符的頻率應具有指數關係，每對相鄰音符之間的比例 r 滿足 $r^{12} = 2$，即 $r = 2^{1/12}$。透過將起始頻率乘以比例 $r = 2^{1/12}$ 12 次，我們就可以獲得八度內所有音符的頻率（表二）。

調性變換

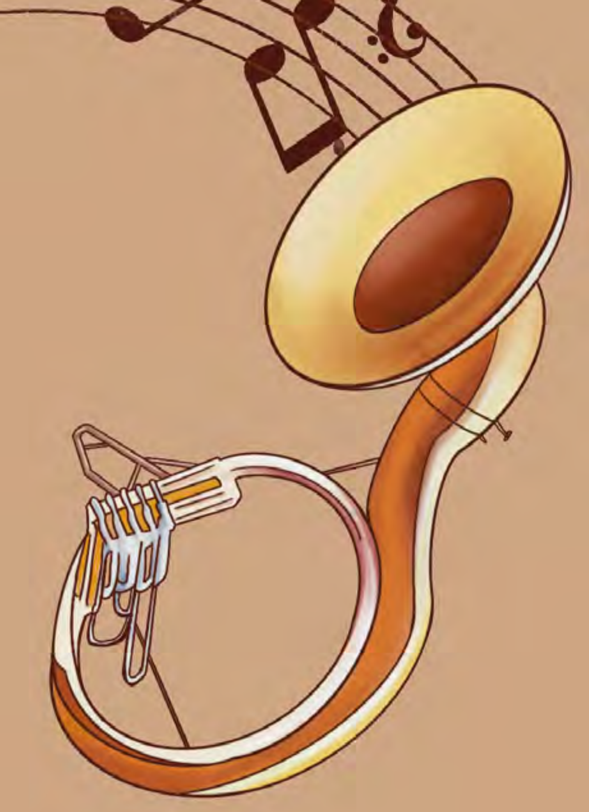
那麼，為甚麼十二平均律比畢氏音律更受青睞呢？你可能聽過一個音樂術語叫「轉調」，它在數學上是指將旋律中每個音符的頻率乘以一個常數，這樣做的話人腦仍會將新旋律視為與舊旋律相同，因為任何兩個相鄰音符之間的音程（即頻率比）保持不變 [1]。例如一段由 440 Hz、660 Hz 和 733.3 Hz 音符組成的旋律聽起來與由 550 Hz、825 Hz 和 916.6 Hz 組成的是同一個旋律。音樂中的調性變換為音樂家提供表達情感的途徑：在樂曲中途轉換到更高調性

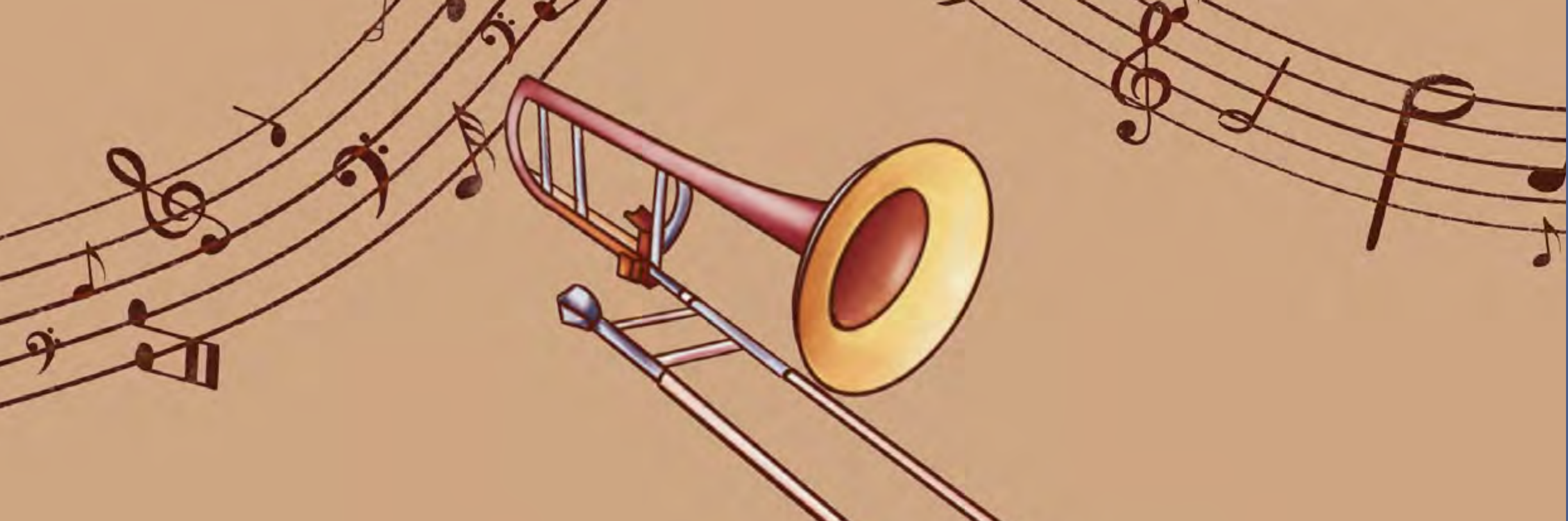
比例	1	9/8	81/64	4/3	3/2	27/16	243/128	2
頻率(Hz)	440	495	557	587	660	743	835	880

表一 畢氏音律中各音符的頻率以及它們與 A 音頻率之比。數值四捨五入至最接近的整數。

比例	1	$2^{1/12}$	$2^{2/12}$	$2^{3/12}$	$2^{4/12}$	$2^{5/12}$	$2^{6/12}$
頻率(Hz)	440	466	494	523	544	587	622
比例	$2^{7/12}$	$2^{8/12}$	$2^{9/12}$	$2^{10/12}$	$2^{11/12}$	2	
頻率(Hz)	659	698	740	784	831	880	

表二 十二平均律中各音符的頻率以及它們與 A 音頻率之比。數值四捨五入至最接近的整數。表中棕色方格的頻率在鋼琴中由黑鍵演奏。





可以表達振奮和鼓舞，而降低調性則可以傳達悲傷或平靜的感覺。此外，通過降低歌曲的調性，音域較低的歌手也能夠演唱原本包含高音的歌曲。

了解調性變換的概念後，你會發現轉調能完美地在十二平均律中執行，因為任何相鄰音符之間的頻率比是一個常數 [1, 2]，譬如鋼琴手只需在以十二平均律調音的鋼琴上將旋律中每個音符順移一個鍵，就能完成調性變換，而鍵盤上有限數量的鍵足以涵蓋調性變換所需的任何音符。

另一方面，畢氏音律中的七個音符並不足夠進行調性變換，因為相鄰音符之間的頻率比並不恆定，可以是 9:8 或 256:243 [2]。我們必須延續畢達哥拉斯的計算以創造出更多的音符，才能從任何音符起進行調性變換。透過將他的計算擴展到第一個八度之外，我們希望頻率會在某處返回起點的 440 Hz，這樣我們就不會製造出無限多的音符。然而，這已被證明是不可能的，因為 $(3/2)^n$ 永遠不會是二的次方數，因此我們需要製造無限多的黑鍵來讓樂器進行調性變換，但這是不切實際的 [1]。雖然畢達哥拉斯的計算在某處已非常接近所需的頻率 440 Hz，但仍存在一個被稱為「畢氏音差」的小差距 [2]。這個略高於一的頻率比 1.0136:1 使音樂家和數學家苦惱不已，直到十二平均律出現為止（註二）。

發明十二平均律的歷史爭議

有個有趣巧合，就是十二平均律由中國數學家、物理學家及音樂理論家朱載堉於 1584 年提出，然後在 1585 至 1608 年間由佛蘭德數學家 Simon Stevin 給予數學上的定義 [3]。對於二人當中是誰發明了十二平均律，以及輾轉間二人到底是否得知對方研究這件事至今仍然存在爭議 [3, 4]，但也許我們永遠無法得知真相。

儘管如此，我們能從中學到的是今天我們視為理所當然的任何事物，可能都是前人經過幾千年努力所取得的成果，背後可能也有科學上的根據。從畢達哥拉斯的早期研究到十二平均律的發明，數學家塑造了我們今時今日所聆聽的音樂。因此下次當你唱「do-re-mi-fa-sol-la-ti」時，希望你能回想起這些熟悉音符背後的數學故事，感慨音樂世界中的數學之美。

1 中音 A：在 C 大調（音樂中最簡單的調之一），「do-re-mi-fa-sol-la-ti」分別對應 C、D、E、F、G、A、B。作為樂器調音的標準音符，中音 A 對應 C 大調中的「la」。根據 ISO 16 標準，它的頻率應為 440 Hz，但某些管樂團有時為了迎合管樂器會將它調成 442 Hz [5]。

2 編按：1.0136 是由 $3^{12} / 2^{19}$ 所得，即是升 12 次五度和降七次八度。

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[2] Benson, D. (2008, December 14). *Music: A Mathematical Offering*. Cambridge University Press. *https://homepages.abdn.ac.uk/d.j.benson/pages/html/music.pdf*

[3] Yung, B. (1981). *A Critical Study of Chu Tsai-yü's Contribution to the Theory of Equal Temperament in Chinese Music*. By Kenneth Robinson. Additional Notes by Erich F. W. Altwein; Preface by Joseph Needham. Wiesbaden: Franz Steiner Verlag (Sinologica Coloniensia Band 9), 1980. x, 136 pp. Figures, Appendixes, Bibliography. N.p. *The Journal of Asian Studies, 40*(4), *775–776.*

[4] Kuttner, F. A. (1975). Prince Chu Tsai-Yü's life and work: A re-evaluation of his contribution to equal temperament theory. *Ethnomusicology, 19*(2), 163–206.

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John Horton Conway: The Mathematician Who Made Math Fun

John Horton Conway: 讓數學變得有趣的數學家

By Helen Wong 王思齊

Quiz time: Can you recognize the following pattern?
突擊測驗：你能認出以下圖案嗎？

Animation 動畫：

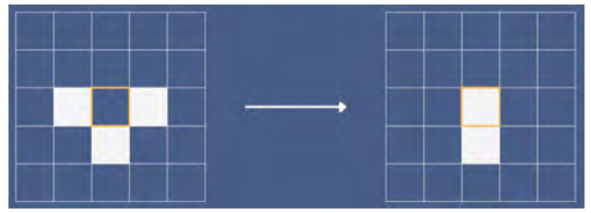
Conway's Game of Life
Conway的生命遊戲

The Number Sequence That Needs to Be Said (Issue 023)
唸出數列之必要：聽射性數列（第二十三期）

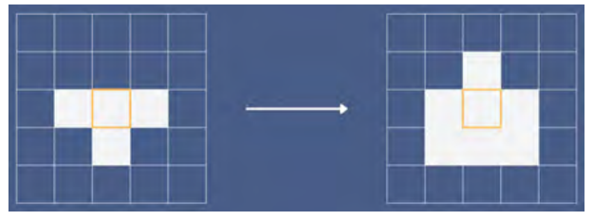
The Game of Life

If you are an avid fan of the Game of Life, you will immediately identify this as the famous Gosper glider gun. However, for those new to the game, you might be surprised to learn that this lifelike animated pattern – a gun periodically firing a stream of gliders – is built upon a deceptively straightforward set of rules [1] (Table 1). The player can start the game by specifying a "seed pattern," which will "evolve" in each round (or "generation") based on the rules. Some hardcore fans have classified the output and created a lexicon with all the special "life forms."

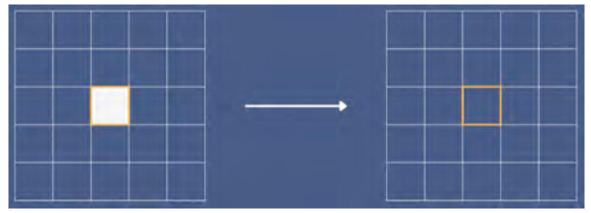
1. **Birth:** A dead cell becomes alive when it has three live neighbors.



2. **Survival:** A live cell survives when it has two or three live neighbors.



3. **Death (underpopulation):** A live cell dies when it has one or no live neighbors.



4. **Death (overpopulation):** A live cell dies when it has four or more live neighbors.

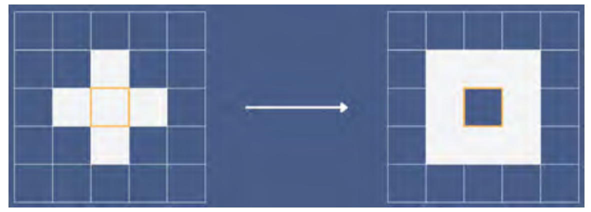


Table 1 The four rules of the Game of Life [1]. Each cell has eight neighbors. Shaded cells are alive while empty cells are dead.

John Horton Conway

For over half a century, the Game of Life has fascinated countless mathematicians, computer scientists, and hobbyists with its infinite possibilities. And its creator, British mathematician John Horton Conway (1937–2020), lived a life just as full of wonder and playfulness.

Born in Liverpool in 1937, Conway's career followed a familiar path of some of the greatest mathematicians of his time: He became interested in mathematics at an early age, went on to study mathematics at the University of Cambridge and received his doctorate there in 1964, then stayed on as a faculty member until moving to Princeton University in 1986 [2, 3].

Yet, on the other hand, Conway was very much an atypical academic [3, 4]. Instead of being confined to his office at Princeton, he enjoyed roaming around the departmental common room and the hallways of the mathematics building. As a teacher, he often brought pocket props (such as ropes, cards, dice, Slinkys, and even a toy bike!) to his classes and would sometimes even "throw a shoe at the window if he thought the students were asleep [4]." Perhaps most famously, he approached mathematics with games and puzzles.

Month	January	February	March	April	May	June
Doomsday	3/1 (or 4/1 on a leap year)	28/2 (or 29/2 on a leap year)	14/3	4/4	9/5	6/6
Month	July	August	September	October	November	December
Doomsday	11/7	8/8	5/9	10/10	7/11	12/12

Table 2 A set of easily memorable doomsday that always fall on the same day of the week. (Can you spot some patterns? Any important date for the hungry mathematicians?)

"I learned very quickly that playing games and working on mathematics were closely intertwined activities for him [John Conway], if not actually the same activity."

— Manjul Bhargava, Professor of Mathematics at Princeton University [3]

Doomsday Algorithm

Here's yet another mathematical trick invented by Conway that you will not only enjoy but also find useful: the Doomsday Algorithm [5]. By working out the "doomsdays," or a set of easily memorable dates that always fall on the same day of the week in the same year (Table 2), you can quickly determine the day of the week for any given date. Before delving into how the algorithm works, let's first clarify what we mean by the term "doomsday." Here, doomsday is defined as the day of the week for any doomsday dates in a year.

Let's start from a basic fact – the doomsday of 2000 was Tuesday. Conceptually, by observing how the doomsday shifts across years and centuries, one can work out the doomsday of any year. We can then find the day of the week of a particular date by calculating the difference between the date and its nearest doomsday date.

Still confused? Let's go over a simple example together [5]. Here's a tip: You can always refer to the cheat sheet at the end of the article for quick reference to any "unfamiliar numbers" you find in the next paragraph.

Suppose we want to know the day of the week for December 26, 1937 (Guess what? It's Conway's birthday! [2]). First, let's start from year 1900. The century code for the 1900s is 3, meaning that the doomsday of 1900 was Wednesday. Here's a fact: The day of the week of the same date moves forward by one day in the next year, and by two days for a leap year. Conway has, again, analyzed the pattern of the results and developed a "year shortcut." For the year 1936, we can add the year shortcut 3 to the century code, so 3 + 3 = 6. The doomsday of 1936 was Saturday. Then we may safely arrive at the conclusion that the doomsday of 1937 was Sunday, as there was no leap day after December 26, 1936 until the date of interest.

The nearest doomsday date to December 26 is December 12, so we subtract 12 from 26 to get 14. Dividing 14 by 7 gives us a remainder of 0, meaning that December 26 falls on the same day of the week as the doomsday of 1937, i.e. Sunday.

Voila! We now know that Conway was born on a Sunday.

While most may only use this as a party trick, Conway took it to another level. He challenged himself to calculate ten random dates to log on to his computer and set a goal of doubling his speed every five years [6]. His record was an impressive 15.92 seconds!

Audioactive Sequences

By now, some of you may recall that we previously covered another piece of Conway's recreational mathematics work in Issue 023: audioactive sequences, in which the next term in a series is generated by reading aloud the preceding term (Footnote 1). Conway discovered some intriguing properties of these sequences and described them with chemistry jargon, but we won't delve into that here to avoid spoilers for those who have not read the article.

"Serious Mathematics"

Although Conway's name is often associated with mathematical games and puzzles, most notably the Game of Life, it would be unjust to overlook his significant contribution to numerous branches of "serious mathematics" including group theory, higher-dimensional geometry, tessellations, knot theory, number theory, abstract algebra, mathematical logic and analysis [7]. In fact, Conway himself considered his invention of the "surreal numbers" to be his greatest achievement [3, 4]. This new number system encompasses all real numbers, including integers, fractions, and irrational numbers, as well as "infinitely small" and "infinitely large" numbers. As was often the case, Conway's inspiration for the surreal numbers came from his contemplation of the rules of a game. It was the game of Go on this occasion.

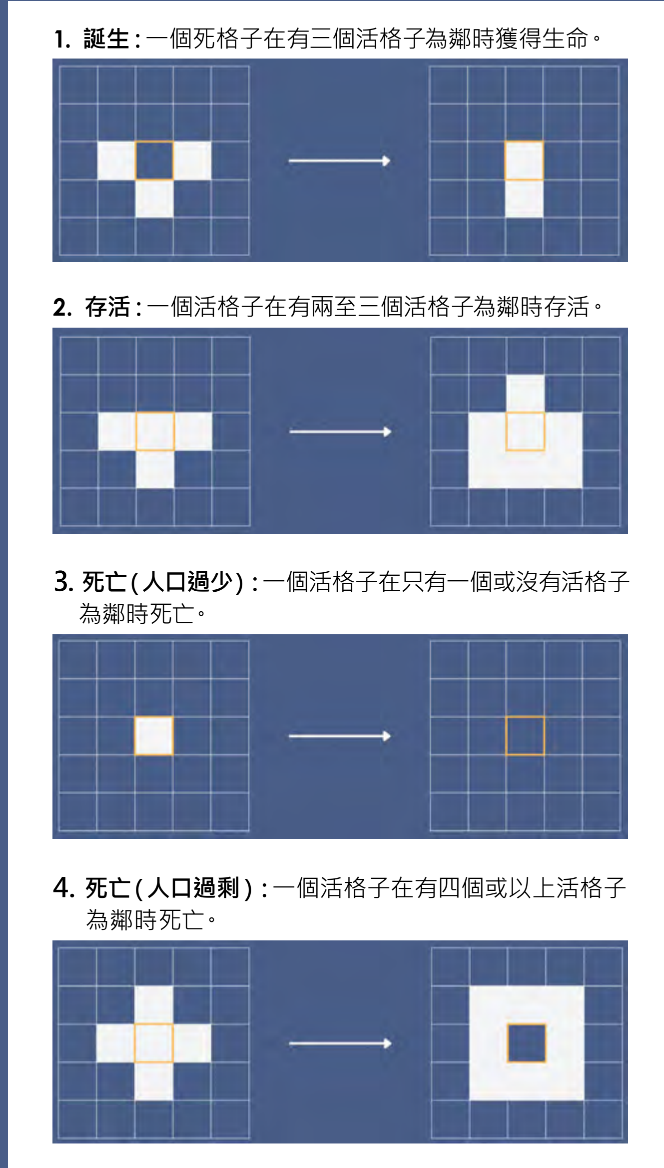
Concluding Remark

Whether it was to the mathematical community, game enthusiasts worldwide, or the Princeton campus, Conway's passing due to COVID-19 complications in 2020 was a great loss. Nonetheless, his spirit of viewing mathematics as a form of play continues to inspire. This enduring legacy is perhaps best exemplified by the recent discovery in the Game of Life of the two long missing repeating patterns that repeat itself after 19 and 41 "generations" respectively [1].

1. Editor's note: For example, the sequence 1, 11, 21, 1211, … is an audioactive sequence which can be read as "one 1, two 1's, one 2 one 1, …" Similar to the Game of Life, Conway and math lovers seriously investigated the pattern of numbers emerged from the puzzle.

生命遊戲

如果你是生命遊戲(The Game of Life)的資深愛好者,定能一眼認出這就是著名的「高斯帕滑翔翼機關槍」(Gosper glider gun)。但對新手來說,你可能會驚訝地發現這個栩栩如生的動畫效果(不斷發射滑翔機的機關槍)竟僅建基於一組看似簡單的規則 [1](表一)。玩家開始遊戲前需指定一個「起始圖案」,該圖案將根據規則在每個回合(或稱為「世代」)「進化」。一些忠實粉絲對遊戲產生的圖案進行了分類,彙編了包含所有特殊「生命形態」的圖鑑。



表一 生命遊戲的四個規則 [1]。每格均有八個「鄰居」。灰色方格代表活格子,藍色方格則代表死格子。

John Horton Conway

半個多世紀以來，生命遊戲以其無限可能性讓無數數學家、電腦科學家和業餘愛好者為之著迷，而它的創造者，英國數學家 John Horton Conway（1937-2020）的一生也是充滿奇思妙想和無窮樂趣。

1937 年出生於利物浦的 Conway 和同時代的一些偉大數學家在事業上踏著似曾相識的路：他自小便對數學產生濃厚興趣，然後進入劍橋大學學習數學，並於 1964 年獲得該校的博士學位，隨後留校任教至 1986 年再轉至普林斯頓大學 [2, 3]。

然而，Conway 的行事作風卻絲毫不像一個典型的學者 [3, 4]。他不喜歡被局限在普林斯頓大學的辦公室裡，而是喜歡在系裡的休息室和數學大樓的走廊遊走。作為教授的他經常帶著一些小道具（例如繩子、卡片、骰子、彈簧，甚至是玩具自行車！）去上課，有時「如果他認為學生在打瞌睡的話，就會把一隻鞋子扔向窗戶 [4]」。也許最為人津津樂道的，是他透過遊戲和謎題來研究數學。

「我很快便發現對於他〔John Conway〕來說，玩遊戲和研究數學密不可分，彷彿就像同一個活動。」

— 普林斯頓大學數學教授 Manjul Bhargava [3]

讓我們從一個簡單事實開始 — 2000 年的末日都落在星期二。理論上透過觀察末日在不同年份和世紀裡落在星期幾的規律，就可以歸納出能計算任何一年末日是星期幾的方法。然後，透過計算某個特定日期與其最接近的末日日期之間的差值，便能找出該特定日期是星期幾。

還是感到困惑嗎？那讓我們看看一個簡單例子 [5]。提提你，如果在下文看見任何不知從何而來的數字，你可以隨時翻看文末的備忘錄。

假設我們想知道 1937 年 12 月 26 日是星期幾 — 猜猜看？那是 Conway 的生日！[2]）— 我們可先從 1900 年入手。20 世紀的代碼是 3，意味著 1900 年的末日是星期三。此處要說明一個事實：同月同日的日子在下一年會在一週裡順移一天，即是例如星期一會變成星期二，而在閏年則會順移兩天。Conway 又再一次分析了計算結果的規律，並總結成「年份捷徑」。就 1936 年而言，我們可以將年份捷徑 3 加到世紀代碼上，即 3 + 3 = 6。因此，1936 年的末日是星期六。然後我們可以從推論得知 1937 年的末日是星期日，因為從 1936 年 12 月 26 日到我們的目標日期之間沒有閏日。

月份	一月	二月	三月	四月	五月	六月
末日	3/1（在閏年則是 4/1）	28/2（在閏年則是 29/2）	14/3	4/4	9/5	6/6
月份	七月	八月	九月	十月	十一月	十二月
末日	11/7	8/8	5/9	10/10	7/11	12/12

表二 一組容易記住，而且總是落在一星期中同一天的末日日期（你能從中發現一些規律嗎？對飢腸轆轆的數學家而言，哪個日子別具意義？）

末日算法

這裡要介紹另一個由 Conway 發明，有趣又實用的數學把戲：末日算法（Doomsday Algorithm）[5]。透過利用一組容易記住，而且每年總是落在一星期中同一天的「末日」（表二），你就能快速計算出任何一個日子是星期幾。

最接近 12 月 26 日的末日是 12 月 12 日，所以我們將 26 減去 12，得出 14，然後將 14 除以 7，得出餘數為 0，意味著 12 月 26 日與 1937 年的末日都是一週裡的同一天，亦即是星期日。

太好了，我們現在知道 Conway 在星期日出生了！

雖然大多數人可能只將末日算法視為一個派對把戲，但 Conway 並不會就此罷休。他決定挑戰自己，逼自己在登入電腦時必須計算十個隨機日期，又定下每五年將速度提高一倍的目標 [6]。因此他的最佳紀錄是令人嘆為觀止的 15.92 秒！

聽射性數列

讀至這裡，相信有讀者已經想起我們在第二十三期介紹過 Conway 的另一個數學玩意：聽射性數列（audioactive sequences）。這種特殊數列裡的每個項都是透過朗讀前一個項而產生的（註一）。Conway 發現了這種數列的一些有趣特性，並以化學術語加以描述，但為了避免對還未讀過那篇文章的讀者劇透，我們不會在此深入探討這種數列。

「正經」數學

儘管 Conway 的名字經常與數學遊戲（尤其是生命遊戲）以及謎題聯在一起，但我們絕不能忽視他在眾多「正經」的數學範疇上的重大貢獻，譬如群論、高維幾何、密鋪、紐結理論、數論、抽象代數、數理邏輯與分析等 [7]。事實上，Conway 本人認為自己最偉大的成就是提出了「超現實數」（surreal numbers）的概念 [3, 4]。這個新的數字系統涵蓋了所有實數，包括整數、分數和無理數，以及「無限小」和「無限大」的數字。一如以往，Conway 對「超現實數」的靈感來自於他對遊戲規則的思考 — 這次啓發他的是圍棋。

結語

Conway 在 2020 年因 COVID-19 併發症去世對無論是數學界、全球遊戲愛好者，還是普林斯頓大學都是一大損失。儘管如此，他將數學視為遊戲的精神依然存在，並持續啓發著人們，當中最能體現這種延續的一例，或許就是最近在生命遊戲中發現的兩個長久以來也沒有人能找到重複圖案，它們分別會在 19 和 41 個「世代」後由頭重複一遍 [1]。

1 編按：例如數列 1, 11, 21, 1211...... 便是一個聽射性數列，它可以讀作「一個一；兩個一；一個二，一個一 ......」。與生命遊戲一樣，Conway 和數學愛好者認真研究了這種特殊數列中數字的規律。

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Cheat Sheet of the Doomsday Algorithm
末日算法備忘錄

The day of a week 日子	Sunday 星期日	Monday 星期一	Tuesday 星期二	Wednesday 星期三	Thursday 星期四	Friday 星期五	Saturday 星期六
Code 代碼	0	1	2	3	4	5	6

Table 3a Numerical representation of the days of a week.
表三甲 一星期中每天的數字代碼

Year 年份	1700	1800	1900	2000
The day of a week of the doomsdays 末日落在的日子	Sunday (0) 星期日 (0)	Friday (5) 星期五 (5)	Wednesday (3) 星期三 (3)	Tuesday (2) 星期二 (2)
Year 年份	2100	2200	2300	2400
The day of a week of the doomsdays 末日落在的日子	Sunday (0) 星期日 (0)	Friday (5) 星期五 (5)	Wednesday (3) 星期三 (3)	Tuesday (2) 星期二 (2)

Table 3b Century code from the year 1700 to 2400. In fact, the century code "0, 5, 3, 2, ..." repeats every four centuries. In other words, any two centuries share the same code if they differ by a multiple of 400 years.
表三乙 從 1700 年到 2400 年的世紀代碼。事實上，世紀代碼「0、5、3、2 ......」每四個世紀就會重複一次。換句話說，任何兩個世紀只要相差 400 的倍數，它們的世紀代碼都是一樣的。

The last two digits of the year 年份的最後兩位數字	00	12	24	36	48
Value 對應數值	0	1	2	3	4
The last two digits of the year 年份的最後兩位數字	60	72	84	96	
Value 對應數值	5	6	7	8	

Table 3c Year shortcut.
表三丙 年份捷徑

Q&A with Three Generations of HKUST Math Majors
讀數學系的人：與三代科大學子對談

By Aastha Shreeharsh

Uras SOMER
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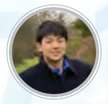
本校理學教育助理教授

What inspired you to major in mathematics?
甚麼啟發你選擇主修數學？

Victor:

I don't think there was one, but I was always interested in math — in solving a question by logical sense. When I started thinking about what I can study in university, mathematics was quite attractive. When I was doing past papers for A-levels, I was obsessed with doing the ones for mathematics. After not so much thought and consideration, I thought, why not? I really wanted to do mathematics instead of giving presentations, although at this stage in my career I can understand just how important presentation skills are, especially in the industry. I was very happy when I solved every single math question at that point. When I was choosing the stream of study — I need to emphasize that I still loved math so much — I chose the very serious mathematics, that was pure mathematics. But if I could go back in time, I would have picked applied mathematics.

也許沒有一個特別原因，但我一直都對數學感興趣 — 對以邏輯思維解決問題感興趣。當我開始考慮在大學主修甚麼時，數學是個頗具吸引力的選擇。操練高級程度會考試題時，我對寫數學試題特別著迷。在沒有顧慮太多的情況下，我想：有何不可呢？我真的想做數多於做匯報，儘管事業到了這個階段我完全理解演講技巧在商業上有多重要。那時我每解決一道數學題都會感到非常興奮。選擇學習方向時 — 我必須強調我仍然非常喜歡數學 — 我選擇了非常「硬派」的數學：純粹數學。但如果我能回到過去，我會選擇應用數學。

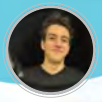


Anson:

It was not a single person or event that sparked my interest in mathematics. I suppose I was genuinely interested in science during my secondary school time, and to learn science, you need a little bit of math. I remember reading something like "Calculus in Comics" and had some interest in math early on. Yet, I was more interested in chemistry in Form 4, but the more I learned about chemistry, the more I realized things are not so absolute. There are always exceptional cases, but memorizing every exception was frustrating — thus, I turned to math.

並沒有特定一個人或一件事激發我對數學的興趣。我想我在中學時期就真正熱愛科學，而學習科學需要一點數學。我記得讀過「漫畫中的微積分」之類的書，並很早就對數學產生興趣。然而，在中四時我對化學更感興趣，但當我知道越多，就越發現內容並不是那麼絕對，總會出現例外的情況，但要不停記住例外的例子令人沮喪 — 因此，我轉向了數學。

Did you ever consider any other major or career?
你考慮過其他主修或職業嗎？



Uras:

Honestly, I was pretty lost about my career when I was younger. I never seriously considered any other major or career path — maybe, I was very interested in astrophysics for a while; but once that spark of mathematics came, I did not really change paths for a long time — perhaps since the beginning of high school. For me, now I see, there is no way I could have done physics — obviously, physics has a close relationship with math but they are also incredibly different. I think the separating line is whether you are fine with just thinking and doing something for the sake of things; you really have to enjoy mathematics for the sake of it. I really liked how perfect mathematics is; in physics, we build on our observations rather than proofs and it left me unsatisfied. Different tastes.

老實說，小時候我對自己的未來路向感到迷茫，但此後我幾乎沒有考慮過其他主修或職業路向 — 也許我曾對天體物理學非常感興趣，但在邂逅數學後，就沒有真正改變過方向，可能是從剛開始讀高中的時候就一直是這樣吧。現在我才明白到，我根本不可能讀物理 — 物理顯然跟數學有密切關係，但它們非常不同。我認為分界線在於你是否能接受僅僅為了事情本身而思考和做一件事情；你必須純粹為了數學本身而享受數學。我很喜歡數學的完美；物理學建基於觀察而不是證明這點不能滿足我。這是兩種不同的取向吧。

Dr. Leung 梁博士：

Especially at the time of my master's program, I thought of working towards becoming an investment banker after doing a master's degree in financial mathematics, but I decided against it due to the inflexible working hours. Personally, I do not really like that lifestyle, despite the significant amount of money you can earn. At that time, I liked teaching more. I remember when I chose the math major at HKUST, I set a goal for myself one day to become a lecturer or professor at a university. I'd say it's a very flexible life, and you have more freedom which I prefer. I chose to stay in academia instead of going into the industry.

特別是讀碩士的時候，我曾考慮在修讀金融數學碩士後成為一名投資銀行家，但最終因為不能靈活調動工作時間而決定放棄。就我個人而言，儘管可以賺到相當多金錢，但我並不喜歡那種生活方式。那時我更喜歡教學工作。還記得在科大選擇主修數學時，我給自己定了一個目標，就是希望有天能成為大學講師或教授。我會說這是一種時間安排上比較靈活的生活，亦可享有更多自由，這是我所偏好的。因此我選擇留在學術界，而不是進入商界。

Sonia:

Originally, in high school, I considered going into music because I played piano very seriously. I decided against that as I was also very interested in the sciences. At the time, I was very interested in particle physics. My dream was to work at CERN, the particle accelerator, do cool things and discover new particles. What attracted me to physics was more the theory side — which sort of explains why I went into math instead. When you read pop science books for the first time and come across the Grand Unified Theory at age 12, you think you really want to do that and change the world. I would watch Neil deGrasse Tyson, MinutePhysics, and a bunch of channels like 3Blue1Brown and Numberphile, which sustained my interest in the science and math side of things. I came into the School of Science with my major undeclared, but after physics lab courses, I realized it wasn't for me because I kept breaking things in the lab. I'm quite sure my lab technician will remember me for years to come.

最初在高中時，我考慮過選擇音樂，因為我一直非常認真對待鋼琴演奏。然而我最終決定放棄，因為我對科學也很感興趣。那時我對粒子物理學著迷，夢想是在 CERN 那個粒子加速器工作，幹一些了不起的事情，發現一些新粒子。使我對物理學產生興趣的，更多是在理論那端 — 這也解釋了為甚麼我最終選擇數學。當你 12 歲那年第一次閱讀科普書籍，接觸到大一統理論時，你會想，你真的想把這個問題解決從而改變世界。我會收看 Neil deGrasse Tyson、MinutePhysics，以及一些像 3Blue1Brown 和 Numberphile 的頻道來維持我對科學和數學方面課題的興趣。我進入理學院時尚未選擇主修科目，但在物理實驗課後，我意識到物理並不適合自己，因為我在實驗室不斷打破東西。我肯定實驗室技術員以後都不會忘記我。

If you could change one thing about how mathematics is taught in schools, what would it be?
如果你可以改變學校裡數學教學方式中的其中一環，你會改變甚麼？

Victor:

I would say the more group projects you work on, the more you can develop yourself in different respects. Not just only focusing on the problems themselves, you can develop the problem by yourself, solve that problem with other people, and learn how to work with them. I think collaboration and communication skills can be developed earlier in life — when some math students are applying for jobs, they have not been trained that intensively to talk to people professionally as business students were. So if I could choose, I would rather prefer more group projects or even opportunities to just present your ideas in front of the class.

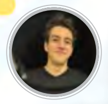
我認為參與更多小組作業能更有助不同方面的成長。除了專注於解決問題本身，你還可以自己一步步提出問題，然後與組員一起解決問題，並學習如何與組員合作。我認為可以在學生時期就開始發展合作和溝通技巧；在求職時，一些數學畢業生並沒有如商科學生那樣接受過密集訓練去學習職場溝通技巧，因此如果我可以選擇，我會希望有更多小組作業，或者只是在班上分享想法的機會也好。

Sonia:

At least in Hong Kong, since I came from the local school system, the way math is taught here is very sort of rigid. People often rely on past papers to get "good at math" but college math does not work that way. Also, there are sometimes, arbitrary cuts to the syllabus that occur at the most random times, and it doesn't really give a full picture of what math is really like. I'm not an expert in education, but I think maybe introducing more options for students in senior secondary school would be good. Right now, the situation is that a bunch of students will be asked to understand the math they will never use, such as 3D geometry projection problems, since they would study history or law; on the other hand, there are students who want to go into engineering or mathematics-heavy degrees and they clearly need more math background. So, introducing more options would be most beneficial. For example, in the extended math curriculum, integration is taught but not integration of trigonometric functions fully.

至少在香港，因為我來自本地學校，這裡教授數學的方式可以說是非常僵化。歷年試題經常被用於證明一個人「精於數學」，但大學裡的數學並不是這回事。此外，課程中的不同地方時常會被不明所以地隨意刪減，使中學課程不能反映數學全貌。雖然我不是教育專家，但我認為可以在高中階段為學生提供更多選擇。現在的情況是許多學生被要求理解一些他們永遠不會用到的數學，例如準備學習歷史和法律的同學需要理解三維幾何投影問題；另一方面，一些想修讀工程或與數學密切相關學位的同學顯然需要更深入的數學背景。因此，提供多元選擇將帶來最大益處；例如延伸數學課程中雖然涵蓋了積分，但並沒有完全包含三角函數的積分。

Do you have any advice for high school students thinking of studying math?
你有甚麼建議給予正在考慮修讀數學的高中生？



Uras:

Just make sure you really love what you are doing. Make sure you are not bound by other factors too heavily — perhaps, like chasing money, which might be difficult especially if you want to be a pure mathematician; although, if that's your passion, you can study it first and branch off to many other career paths other than being a professor. Be prepared to face challenges. If you are stuck on a problem, don't give up and give it time. The answer might come to you after a good night's sleep or on the MTR; it's not the end of the world if it doesn't come to you immediately. Don't be afraid to seek help, ask your peers or teachers. There's no shame in seeking hints or help, as long as you understand the answer.

你需要確定自己真正熱愛你所做的事情，並確保你不受其他因素束縛太多 — 例如追逐金錢，這可能會很困難，尤其如果你想成為純粹數學家；雖然如果那是你的熱情所在，你大可以先讀純數，然後轉向教授以外的許多其他職業。要準備好面對挑戰，如果你卡在某個問題上，不要放棄，給自己一點時間。答案可能會在你好好睡一覺後，或某天坐港鐵時出現；即時想不出答案並不是世界末日。不要害怕尋求幫助，向你的同學或老師詢問。尋求提示或協助並不丟臉，只要找到答案就好。

Dr. Leung 梁博士：

Students should know what they will do after graduation. Math can be treated as an art, at the same time it can be treated as a tool in solving real life problems. For some students who love math as an art and really want to stay in academia, of course you can study pure math and that's it. Let's say if you are not very interested in math, you just want to work in industry — then you need to be careful in choosing math as a major. The most important thing is you need to think about what kind of courses you want to take — you'd want to pick courses in the vein of applied mathematics. You need to pick a track or program that offers that sort of applied mathematics. The second thing is to spend time thinking about why certain math concepts are important. You need some time to understand the motivations and basics of a concept in order to solve problems effectively, especially if you are not pursuing math for the sake of it, or viewing math as an art.

學生應該知道畢業後的打算。數學可以被視為一種藝術，同時也可以作為解決現實問題的工具。把數學當作藝術並衷心熱愛數學，又想留在學術界的同學當然可以修讀純粹數學，這是正確選擇。假設你對數學不是很感興趣，只想進入外頭的相關行業 — 那麼在選擇數學作為主修時你需要謹慎考慮。最重要的是，你需要考慮選甚麼樣的課 — 你會想選應用數學類的課。你需要選擇提供這類課的課程。第二件事是必須花時間思考某些數學概念為何重要，你需要理解一個概念背後的動機和基礎，才能有效地解決問題，特別是如果你學習數學並不是為了數學本身，沒有將數學視為一種藝術的時候。

Anson:

Stay interested, and fearless — don't think certain math is for university so you shouldn't study it. There's no such thing as "university math" or "PhD math". Just read what you want to read and persevere, even if you think it is hard. You don't have to be discouraged or disappointed if you don't understand now. Sometimes, even after exams, I couldn't thoroughly understand the concepts but a few months later, it became clear when you pick it up again. You don't have to force yourself to complete a whole book within a month or so, you can come back anytime and it sometimes works better that way.

保持興趣，不要畏懼 — 不要因為認定某些數學是大學內容就不去學習，其實根本沒有所謂「大學數學」或「博士數學」。閱讀你想閱讀的，無論多難都好，堅持下去。即使你現在不能理解，也不必氣餒或失望。有時即使在考試後，我也無法透徹理解一些概念，但幾個月後當你再次接觸這些概念，它們就會開始變得一清二楚。因此你不必強迫自己在一個月讀完整本書，你可以隨時回來繼續，有時這樣做會更有效。

Many students may be interested but intimidated by math. Were there any resources you found helpful during your study?
許多學生對數學感興趣的同時，又可能被數學嚇怕。你有沒有任何有用的學習資源可以分享？

Dr. Leung 梁博士：

If you cannot solve a problem, it's usually just because you do not have enough experience yet. If starting a problem is intimidating, it is just that you need more experience — sometimes, textbooks only give you solutions, instead of how to solve the problem. In that case, the classroom experience is very important! Being able to interact with classmates and teachers is crucial, so that's why I stay behind in class, so students can interact with me and ask me questions without the pressure of interrupting the flow of my lecture.

如果你無法解答一條問題，通常只是因為你還未有足夠經驗。如果嘗試解答問題讓你感到畏懼，那只是因為你需要更多經驗 — 有時教科書只給你答案，但沒有提及如何解答問題。在這種情況下，上課非常重要！能夠與同學和老師互動至關重要，這就是為甚麼下課後我會留在課室，這樣學生就可以與我交流，隨便問而不必擔心打斷課堂。

Sonia:

I guess what got me interested in math was many very good YouTube channels that cover math topics. For example, 3Blue1Brown gives very good visual intuition for linear algebra. Numberphile has some videos on interesting numbers. There are many, many videos about complicated concepts that they try to explain in a more digestible way — however, I do not have a very accurate idea of whether it is digestible for the average person since I'm at a point where once I see a concept, I tend to understand. In my undergraduate years, I would also spend a lot of time on the website Mathematics Stack Exchange when confused with homework. I think another good resource is talking to other people in math — talking to other people who share the same love for math is a great way to further your understanding of math.

我想讓我對數學產生興趣的是許多涵蓋數學不同主題的優秀 YouTube 頻道，例如 3Blue1Brown 擅於把線性代數視覺化，Numberphile 有時會介紹一些有趣數字。那裡還有多不勝數嘗試深入淺出地解釋複雜概念的影片 — 雖然我不肯定這些內容對普通人來說是否也是易於理解，因為我已經到了一個階段，看到概念後往往就能理解。在本科生涯裡面，當我對功課感到困惑時，我會花很多時間在 Mathematics Stack Exchange 網站上，因為我認為與其他數學愛好者交流是另一個很好的「資源」 — 與同樣熱愛數學的人交談有助進一步理解數學。

Visit the following webpage to read the **Complete interview!**
瀏覽以下網頁以閱讀 **完整專訪！**